Defeasible Reasoning over Facts and Norms

Emery Neufeld

TU WIEN Informatics

What are Norms?

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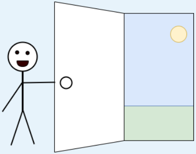

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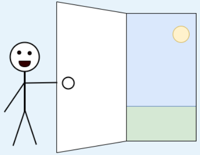

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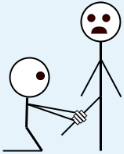

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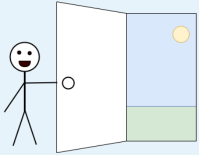

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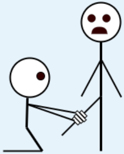

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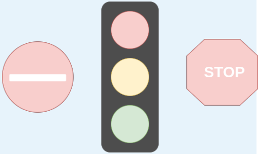

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Constitutive Norms

Constitutive norms are:

- “in context C, X counts as Y"
  - $\mathbf{C}(X, Y|C)$
  - Used to define new concepts.
  - “Eating carrot cake counts as eating cake”.

Normative Systems

Regulative Norms + Constitutive norms = **Normative System**

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Normative System Example

- You are forbidden from eating cake: $\mathbf{F}(cake|\top)$
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Normative System Example

- You are forbidden from eating cake: $\mathbf{F}(cake|\top)$
- You are permitted to eat carrot cake on Tuesday: $\mathbf{P}(carrot|tuesday)$
- Eating carrot cake counts as eating cake: $\mathbf{C}(carrot, cake|\top)$
- Correct conclusion: you can only eat (carrot) cake on Tuesdays.

Compliance

Factual Detachment

$$\mathbf{O}(p|q) \wedge q \implies \mathbf{O}(p)$$

Compliance to Obligations

- Compliance := not violated
- An obligation $\mathbf{O}(p)$ is violated when $\mathbf{O}(p)$ is true but p is not.

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Permissions cannot be violated!

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  - What happens when compliance is not possible?

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  - "When driving, you ought not swerve into another lane."
  - "If it is to avoid a collision, you ought to swerve into another lane."
- Normative Deadlock
  - What happens when compliance is not possible?
  - E.g., contrary-to-duty obligations:
    - "You ought not kill."
    - "If you kill, you ought to kill gently."

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Definition

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Why Non-monotonicity?

Example

1. Birds can fly (*bird* $\rightarrow$ *fly*)
2. Sparrows are birds (*sparrow* $\rightarrow$ *bird*)

- We can derive: sparrows can fly (*sparrow* $\rightarrow$ *fly*)

Why Non-monotonicity?

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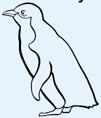
- We can derive: penguins can fly??? (*penguin* $\rightarrow$ *fly*)



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3. **Penguins cannot fly** (*penguin* $\rightarrow \neg$*fly*)

- Is this necessarily a contradiction?

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Literals

$AP :=$ atomic propositions

$Lit := AP \cup \{\neg p \mid p \in AP\}$

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Defeasible Theories

A defeasible theory is a tuple:

$$\langle F, R, > \rangle$$

where $F \subset Lit$ is a set of facts, R is a set of rules, and $>$ is a superiority relation over rules.

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From a defeasible theory, we can derive **conclusions**.

Defeasible Logic (DL): Definite Conclusions

Derived recursively:

Definite Provability

Given a defeasible theory D, if $D \vdash +\Delta p$, then either:

1. p is a fact ($p \in F$), or
2. There is a strict rule r such that:
   1. $N(r) = p$
   2. and for every $a_i \in A(r)$, $D \vdash +\Delta a_i$.

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Definite Refutability

Given a defeasible theory D, if $D \vdash -\Delta p$, then:

1. p is not a fact ($p \notin F$), and
2. For all strict rules r such that $N(r) = p$, it is the case that $\exists a_i \in A(r)$ such that $D \vdash -\Delta a_i$.

Can be computed in linear time!

Defeasible Logic (DL): Defeasible Conclusions (pt 1)

Again, derived recursively:

Defeasible Provability

Given a defeasible theory D, If $D \vdash +\partial p$, either $D \vdash +\Delta p$ or:

1. There is a strict or defeasible rule r such that:
   1. $N(r) = p$ and
   2. for every $a_i \in A(r)$, $D \vdash +\partial a_i$, and
2. $D \vdash -\Delta \neg p$, and
3. For all rules r' such that $N(r') = \neg p$, either:
   1. there is an $a_i \in A(r')$ such that $D \vdash -\partial a_i$, or
   2. There is a strict or defeasible rule r'' such that:
      1. $N(r'') = p$,
      2. for all $a_i \in A(r'')$, $D \vdash +\partial a_i$, and
      3. $r'' > r'$.

Defeasible Logic (DL): Defeasible Conclusions (pt 2)

Defeasible Refutability

Given a defeasible theory D, If $D \vdash -\partial p$, $D \vdash -\Delta p$ and:

1. For all strict and defeasible rules r such that $N(r) = p$ there is $a_i \in A(r)$ such that $D \vdash -\partial a_i$, or
2. $+\Delta\neg p$, or
3. There is a rule r' such that:
   1. $N(r') = \neg p$,
   2. For all $a_i \in A(r')$, $D \vdash +\partial a_i$, and
   3. For all strict or defeasible rules r'' such that $N(r) = p$, either
      1. there is a $a_i \in A(r'')$ such that $D \vdash -\partial a_i$, or
      2. $r'' \not> r'$.

Can be computed in linear time!

Formalizing in DL

Example: the Sparrow

1. Birds can fly. (r_1 : *bird* $\Rightarrow$ *fly*)
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- Suppose we have *sparrow* as a fact. So $+\Delta \mathit{sparrow}$.

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- Then we can derive $+\Delta bird$ from $+\Delta sparrow$ and r_2.

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- Suppose we have *sparrow* as a fact. So $+\Delta sparrow$.
- Then we can derive $+\Delta bird$ from $+\Delta sparrow$ and r_2.
- This means we can derive $+\partial fly$ from $+\Delta bird$ and r_1.

Another attempt at Non-monotonicity

Example: the Penguin (with a defeater)

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- Suppose we have *penguin* as a fact. So $+\Delta \mathit{penguin}$.
- Then we can derive $+\Delta \mathit{bird}$ from r_3 and $+\Delta \mathit{penguin}$
- Then we **cannot** derive $+\partial \mathit{fly}$ from r_1 and $+\Delta \mathit{bird}$; r_4 prevents this.

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- Suppose we have *penguin* as a fact. So $+\Delta penguin$.
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Example: the Penguin (with a superiority relation)

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4. $r_5 > r_1$

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- Then we can derive $+\Delta$*bird* from r_3 and $+\Delta$*penguin*
- Then we derive $+\partial\neg$*fly* from r_5 and $+\Delta$*penguin*; r_1 conflicts, but is defeated.

An Extension: Defeasible Deontic Logic

We can extend DL with deontic operators!

New Syntax

- Introduce modal literals: $ModLit = \{\mathbf{O}(lit) | lit \in Lit\}$
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- $\mathbf{C}(x, y|c)$ translates to $c, x \rightarrow_C y$

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DDL theory = Facts + Normative System

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Violation in DDL

Suppose we have a DDL theory D representing a set of facts F and a normative system. Then a violation is a literal *lit* such that:

$$D \vdash +\partial_O lit,\ -\partial_C lit$$

Example: a Normative System in DDL (pt 1)

A simple normative system...and a violation

- You are forbidden from eating cake. ($r_1 : \Rightarrow_O \neg cake$)
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- There is a violation!

Example: a Normative System in DDL (pt 2)

Adding permission

- Take r_1 and r_2 as above.
- On Tuesdays, you are permitted to eat cake. ($r_3 : \; \mathit{tuesday} \leadsto_O \mathit{cake}$)

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- However, if we also have the fact *tuesday*, r_3 prevents us from deriving $+\partial_O \neg \mathit{cake}$ from r_1.
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- Suppose *gift* is a fact. If we take $r_4 > r_1$, then we derive $+\partial_O cake$ instead of $+\partial_O \neg cake$.
- If we have *carrot* as a fact, we can derive $+\Delta_C cake$ so we cannot derive $-\partial_C cake$.
- No violation!

German

Für das gesamte Plangebiet wird bestimmt: Sofern nichts anderes bestimmt ist, sind Flachdächer von Gebäuden ab einer bebauten Fläche von 30 m^2, soweit sie nicht als begehbare Terrassen ausgebildet werden, nach dem Stand der technischen Wissenschaften zu begrünen.

English

The following is stipulated for the entire plan area: Unless otherwise stipulated, flat roofs of buildings with a built-up area of 30 m^2 or more, unless they are designed as accessible terraces, are to be greened in accordance with the state of the art.

BRISE: Regulation 6817e_5_0

Extracted Concepts

1. BegruenungDach [content] [greened roof]
2. Dachart(Flachdach) [condition] [roof type: flat roof]
3. Dachart(begehbare Terrasse) [conditionException] [roof type: accessible terrace]
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DDL Formalization

$$r_1 : \mathit{GesamtePlangebiet}, \mathit{BebauteFlaecheMin}(30m2), \mathit{Dachart}(\mathit{Flachdach}) \Rightarrow_O \mathit{BegruenungDach}$$

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- Given the above facts, we get $+\Delta_C GesamtePlangebiet$, $+\Delta_C BebauteFlaecheMin(30m2)$, $+\Delta_C Dachart(Flachdach)$, $+\Delta_C BegruenungDach$.

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- Then from r_1 we can derive $+\partial_O BegruenungDach$.
- Since we have $+\Delta_C BegruenungDach$, there is no violation.

Example 2: Facts

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- So both r_1 and r_2 are triggered; r_2 defeats r_1 and we cannot derive $+\partial_O BegruenungDach$.
- There are no obligations to violate.

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- From a set of conclusions, facts, and rules we can always reconstruct the reasoning that led to those conclusions.
- Representing ideal behaviour through rules helps with explainability and transparency.

Literature I

References

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