Answer Set Programming

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15 November 2022

Outline

Introduction

Idea

Design a representation of the world (limited to the small part relevant to the problem at hand) by means of a logic theory; then, solve a problem via automated reasoning on its basis.

Answer Set Programming (ASP)

- **Declarative** programming language for **K**nowledge **R**epresentation and **R**easoning [GL91, EGM97, MT99, Nie99, EFLP00, BET11]
- Logic paradigm based on rules

Answer Set Programming (ASP)

- **Declarative** programming language for **K**nowledge **R**epresentation and **R**easoning [GL91, EGM97, MT99, Nie99, EFLP00, BET11]
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Standard Procedural Programming

- Need for a solving method/algorithm
- Define instructions to be executed “step by step”
- Tell the machine WHAT to do, HOW to solve the problem

ASP-based Declarative Approach

- Specify the features of the desired solution
- NO need for algorithm design
- Just provide the problem specification in form of a logic program
- **Models** will represent solutions
- You just need an ASP solver that computes such models

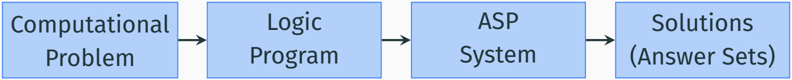
Answer Set Programming (ASP)

- **Declarative** programming language for **K**nowledge **R**epresentation and **R**easoning [GL91, EGM97, MT99, Nie99, EFLP00, BET11]
- Logic paradigm based on rules
- Able to deal with **incomplete knowledge**
- Able to model **non-monotonic reasoning**
  - does not store consequences
  - what deduced up to a certain point can be invalidated after the acquisition of new knowledge

Answer Set Programming (ASP)

- **Declarative** programming language for **K**nowledge **R**epresentation and **R**easoning [GL91, EGM97, MT99, Nie99, EFLP00, BET11]
- Logic paradigm based on rules
- Able to deal with **incomplete knowledge**
- Able to model **non-monotonic reasoning**
- Successfully employed in both academy and industry

Knowledge Representation and Reasoning with ASP



The basic construct of ASP is the one of **rule**:

$$\underbrace{Head}_{\textbf{disjunction}} \text{ :- } \underbrace{Body}_{\textbf{conjunction}} .$$

- Interpreted according to common sense principles
- Roughly, its intuitive semantics corresponds to an implication

```
parent("James Potter","Harry Potter").
son(X,Y) :- parent(Y,X). ⇒ son("Harry Potter","James Potter")
```

ASP Syntax and Semantics

Core Syntax

- An **ASP logic program** is a (finite) set of **rules** of form:

$$\underbrace{a_1 \mid \ldots \mid a_n}_{\textbf{head atoms}} \text{ :- } \underbrace{b_1, \ldots, b_k, \texttt{not } b_{k+1}, \ldots, \texttt{not } b_m.}_{\textbf{body literals}}$$

- A **literal** is either **positive** b or **negative** $\texttt{not } b$ where b is an **atom**
- An **atom** has form $p(t_1, \ldots, t_n)$ (typical FO signature), where:
  - p is a **predicate** of arity n
  - $t_1, \ldots, t_n$ are **terms**
- A **term** is either a **variable** or a **constant**

$\rightarrow \texttt{saga(}\underbrace{\texttt{"Harry Potter"}}_{\texttt{string constant}}\texttt{)}$

$\rightarrow \texttt{numer\_of\_siblings(}\underbrace{\texttt{"Ron"}}_{\texttt{string constant}}\texttt{, }\underbrace{\texttt{6}}_{\texttt{numeric constant}}\texttt{)}$

$\rightarrow \texttt{parent(}\underbrace{\texttt{Y,X}}_{\texttt{variables}}\texttt{)}$

$\rightarrow \texttt{horcrux(}\underbrace{\texttt{ring}}_{\texttt{symbolic constant}}\texttt{)}$

ASP Syntax

Core Syntax

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$$\underbrace{a_1 \mid \ldots \mid a_n}_{\textbf{head atoms}} \texttt{:-} \underbrace{b_1, \ldots, b_k, \texttt{not}\ b_{k+1}, \ldots, \texttt{not}\ b_m}_{\textbf{body literals}}.$$

  - A rule with no literal in its body is a **fact**
  - A rule with no atom in its head is a **strong constraint**
- A program (rule/literal/atom) with no variables is **ground**

```
% fact
saga("Harry Potter").
% (disjunctive) rule
like(X) | dislike(X) :- saga(X).
```

$$\Downarrow$$

```
like("Harry Potter") | dislike("Harry Potter").
```

Core Syntax

- An **ASP logic program** is a (finite) set of **rules** of form:

$$\underbrace{a_1 \mid \ldots \mid a_n}_{\text{head atoms}} \text{ :- } \underbrace{b_1, \ldots, b_k, \texttt{not}\ b_{k+1}, \ldots, \texttt{not}\ b_m}_{\text{body literals}}.$$

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```
%strong constraint
:- pronounce("You-Know-Who").
```

Linguistic Extensions

- Many extensions:
  - new term types: functional terms, arithmetic terms
  - new atom and literal types: aggregate literals, built-in atoms
  - new rule types: weak constraints, choice rules, queries
- Standard input language: **ASP-Core-2** [CFG+12]

```
novel("Harry Potter and the Philosopher's Stone").
novel("Harry Potter and the Chamber of Secrets").
novel("Harry Potter and the Prisoner of Azkaban").
novel("Harry Potter and the Goblet of Fire").
novel("Harry Potter and the Order of the Phoenix").
novel("Harry Potter and the Half-Blood Prince").
novel("Harry Potter and the Deathly Hallows").
num_of_novels(N) :- #count{X:novel(X)}=N. ⇒ num_of_novels(7).
is_not_harry_potter_saga :- num_of_novels(N), N<7.
```

ASP Semantics

ASP semantics is based the concept of answer set [GL91]

Answer sets are only defined for ground programs

- For every non-ground program, a semantically equivalent ground program can be defied
  - **instantiation**, **grounding** process
- A program with variables is just a shorthand for its ground instantiation!

Program Instantiation

Given an ASP program $\mathcal{P}$

- **Herbrand Universe** ($U_{\mathcal{P}}$): set of constants occurring in program $\mathcal{P}$

```
like_herry_potter(X) | dislike_herry_potter(X):- person(X).
person("Pina").  person("Ugo").
```

$\Rightarrow U_{\mathcal{P}} = \{$"Pina", "Ugo"$\}$

Program Instantiation

Given an ASP program $\mathcal{P}$

- Herbrand Universe ($U_{\mathcal{P}}$): set of constants occurring in program $\mathcal{P}$
- **Herbrand Base** ($B_{\mathcal{P}}$): set of ground atoms constructible from $U_{\mathcal{P}}$ and predicates in $\mathcal{P}$

```
like_herry_potter(X) | dislike_herry_potter(X):- person(X).
person("Pina").  person("Ugo").
```

$\Rightarrow U_{\mathcal{P}} = \{$"Pina","Ugo"$\}$

$\Rightarrow B_{\mathcal{P}} = \{$dislike_herry_potter("Pina"),
dislike_herry_potter("Ugo"),
like_herry_potter("Pina"),
like_herry_potter("Ugo"),
person("Pina"), person("Ugo")$\}$

Program Instantiation

Given an ASP program $\mathcal{P}$

- Herbrand Universe ($U_{\mathcal{P}}$): set of constants occurring in program $\mathcal{P}$
- Herbrand Base ($B_{\mathcal{P}}$): set of ground atoms constructible from $U_{\mathcal{P}}$ and predicates in $\mathcal{P}$
- **Instantiation** $ground(\mathcal{P})$: set of the ground instances of rules in $\mathcal{P}$
  - for each rule $r \in \mathcal{P}$: replace each variable in r by a constant in $U_{\mathcal{P}}$

```
like_herry_potter(X) | dislike_herry_potter(X):- person(X).
person("Pina").  person("Ugo").
```

$$\Downarrow$$

```
like_herry_potter("Pina")|dislike_herry_potter("Pina") :- person("Pina").
like_herry_potter("Ugo")|dislike_herry_potter("Ugo") :- person("Ugo").
person("Pina").  person("Ugo").
```

Interpretations

Given an ASP program $\mathcal{P}$ the **Herbrand Interpretation** I for $\mathcal{P}$ is a consistent subset of $B_{\mathcal{P}}$

- an atom a is true w.r.t. I if $a \in I$; otherwise it is false
- a literal `not` a is true w.r.t. I if $a \notin I$; otherwise it is false
- I is consistent if, for each atom a, $\{a, \neg a\} \not\subseteq I$

$$\Downarrow$$

Models

Given an interpretation I:

- I is a model for $\mathcal{P}$ if, for every rule $r \in \mathcal{P}$, whenever the body of r is true w.r.t. I, the head of r is also True w.r.t. I
- I is a *minimal* model for $\mathcal{P}$ if no model N s.t. $N \subset I$ exists

ASP Semantics

```
like_herry_potter(X) | dislike_herry_potter(X):- person(X).
person("Pina").  person("Ugo").
```

$\Downarrow$

```
like_herry_potter("Pina")|dislike_herry_potter("Pina") :- person("Pina").
like_herry_potter("Ugo")|dislike_herry_potter("Ugo") :- person("Ugo").
person("Pina").  person("Ugo").
```

```
I1 = {person("Ugo")} ⇒ (not a model)
I2 = {dislike_herry_potter("Ugo"), person("Ugo"),
      like_herry_potter("Pina"), dislike_herry_potter("Pina"),
      person("Pina")} ⇒ (model, non minimal)
I3 = {dislike_herry_potter("Ugo"), person("Ugo"), person("Pina"),
      like_herry_potter("Pina")} ⇒ (model, minimal)
...
```

ASP Semantics

Reduct

The **reduct** or of a program $\mathcal{P}$ w.r.t. an interpretation I is the program $\mathcal{P}^{\mathcal{I}}$, obtained from $\mathcal{P}$ by

1. deleting all rules with a negative literal false w.r.t. I
2. deleting the negative literals from the bodies of the remaining rules

Answer Set

Given an interpretation I for a program $\mathcal{P}$, I is an **answer set** for $\mathcal{P}$ if it is a minimal model for $\mathcal{P}^{\mathcal{I}}$

$\mathcal{P}$

```
a :- d, not b.
b :- not d.
d.
```

I = { a, d }

a :- d, ~~not b~~.
~~b :- not d.~~
d.

$\mathcal{P}^{\mathcal{I}}$

```
a :- d.
d.
```

$\Downarrow$

I is a minimal model of $\mathcal{P}^{\mathcal{I}}$ and therefore it is an answer set of $\mathcal{P}$

Knowledge Representation and Reasoning with ASP

A Practical Example

Vertex Cover

Given an undirected graph $G = (V, E)$ select $S \subseteq V$ such that all edges are covered (i.e. for every edge $(a, b) \in E$ either $a \in S$ or $b \in S$).

A Practical Example

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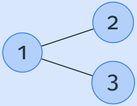
Given an undirected graph $G = (V, E)$ select $S \subseteq V$ such that all edges are covered (i.e. for every edge $(a, b) \in E$ either $a \in S$ or $b \in S$).

1: $node(1).\ node(2).\ node(3).\ edge(1,2).\ edge(1,3).$ $\Rightarrow$ **Facts**
2: $inS(X) \mid outS(X)$:- $node(X).$ $\Rightarrow$ **Disjunctive Rule**
3: :- $edge(X,Y), \texttt{not}\ inS(X), \texttt{not}\ inS(Y).$ $\Rightarrow$ **Strong Constraint**



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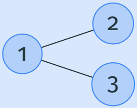
2: $inS(X) \mid outS(X)$:- $node(X).$ $\Rightarrow$ **Disjunctive Rule**
3: :- $edge(X, Y)$, not $inS(X)$, not $inS(Y)$. $\Rightarrow$ **Strong Constraint**



1: $node(1).\ node(2).\ node(3).\ edge(1, 2).\ edge(1, 3).$
2: $inS(1) \mid outS(1)$:- $node(1).$
3: $inS(2) \mid outS(2)$:- $node(2).$
4: $inS(3) \mid outS(3)$:- $node(3).$
5: :- $edge(1, 1)$, not $inS(1)$, not $inS(1)$.
6: :- $edge(1, 2)$, not $inS(1)$, not $inS(2)$.
7: :- $edge(1, 3)$, not $inS(1)$, not $inS(3)$.
8: :- $edge(2, 1)$, not $inS(2)$, not $inS(1)$.
9: :- $edge(2, 2)$, not $inS(2)$, not $inS(2)$.
10: :- $edge(2, 3)$, not $inS(3)$, not $inS(3)$.
11: :- $edge(3, 1)$, not $inS(3)$, not $inS(1)$.
12: :- $edge(3, 2)$, not $inS(3)$, not $inS(2)$.
13: :- $edge(3, 3)$, not $inS(3)$, not $inS(3)$.

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Vertex Cover

Given an undirected graph $G = (V, E)$ select $S \subseteq V$ such that all edges are covered (i.e. for every edge $(a, b) \in E$ either $a \in S$ or $b \in S$).

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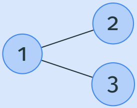

1: $node(1).\ node(2).\ node(3).\ edge(1,2).\ edge(1,3).$
2: $inS(1) \mid outS(1)$:- $\cancel{node(1)}.$
3: $inS(2) \mid outS(2)$:- $\cancel{node(2)}.$
4: $inS(3) \mid outS(3)$:- $\cancel{node(3)}.$
5: $\cancel{:\!-\ edge(1,1), \texttt{not}\ inS(1), \texttt{not}\ inS(1).}$
6: :- $\cancel{edge(1,2)}, \texttt{not}\ inS(1), \texttt{not}\ inS(2).$
7: :- $\cancel{edge(1,3)}, \texttt{not}\ inS(1), \texttt{not}\ inS(3).$
8: $\cancel{:\!-\ edge(2,1), \texttt{not}\ inS(2), \texttt{not}\ inS(1).}$
9: $\cancel{:\!-\ edge(2,2), \texttt{not}\ inS(2), \texttt{not}\ inS(2).}$
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12: $\cancel{:\!-\ edge(3,2), \texttt{not}\ inS(3), \texttt{not}\ inS(2).}$
13: $\cancel{:\!-\ edge(3,3), \texttt{not}\ inS(3), \texttt{not}\ inS(3).}$

A Practical Example

Vertex Cover

Given an undirected graph $G = (V, E)$ select $S \subseteq V$ such that all edges are covered (i.e. for every edge $(a, b) \in E$ either $a \in S$ or $b \in S$).

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2: $inS(X) \mid outS(X) \text{ :- } node(X).$ ⇒ **Disjunctive Rule**
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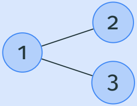

1: $node(1).\ node(2).\ node(3).\ edge(1,2).\ edge(1,3).$
2: $inS(1) \mid outS(1).$
3: $inS(2) \mid outS(2).$
4: $inS(3) \mid outS(3).$
5: $\text{ :- } \texttt{not}\ inS(1),\ \texttt{not}\ inS(2).$
6: $\text{ :- } \texttt{not}\ inS(1),\ \texttt{not}\ inS(3).$

A Practical Example

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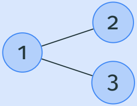


1: $node(1).\ node(2).\ node(3).\ edge(1,2).\ edge(1,3).$
2: $inS(1) \mid outS(1).$
3: $inS(2) \mid outS(2).$
4: $inS(3) \mid outS(3).$
5: :- $\texttt{not}\ inS(1),\ \texttt{not}\ inS(2).$
6: :- $\texttt{not}\ inS(1),\ \texttt{not}\ inS(3).$

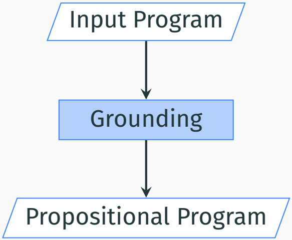
$$AS_1 = Facts \cup \{inS(1), outS(2), outS(3)\} \qquad S = \{1\}$$
$$AS_2 = Facts \cup \{outS(1), inS(2), inS(3)\} \qquad S = \{2,3\}$$
$$AS_3 = Facts \cup \{inS(1), inS(2), outS(3)\} \qquad S = \{1,2\}$$
$$AS_4 = Facts \cup \{inS(1), outS(2), inS(3)\} \qquad S = \{1,3\}$$
$$AS_5 = Facts \cup \{inS(1), inS(2), inS(3)\} \qquad S = \{1,2,3\}$$

Canonical approach to solve an ASP program P over a set of facts F:

ASP Computation

Canonical approach to solve an ASP program P over a set of facts F:

1. **Grounding** (or **Instantiation**) phase:
    - Produces a semantically equivalent ground (i.e., propositional) program: $ground(P \cup F) \subseteq ground_{Herbrand}(P \cup F)$

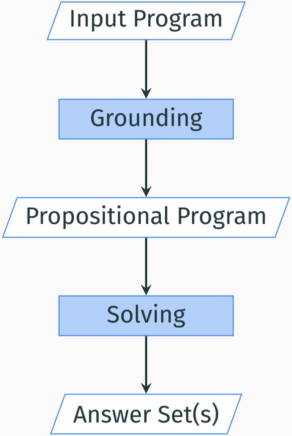


ASP Computation

Canonical approach to solve an ASP program P over a set of facts F:

1. **Grounding** (or **Instantiation**) phase:
   - Produces a semantically equivalent ground (i.e., propositional) program: $ground(P \cup F) \subseteq ground_{Herbrand}(P \cup F)$
2. **Solving** phase:
   - Generates the Answer Set(s) $AS(P \cup F)$

$$AS(P \cup F) \equiv AS(ground(P \cup F)) \equiv AS(ground_{Herbrand}(P \cup F))$$



ASP Systems

Canonical approach: Ground&Solve

- Stand-alone grounders: LPARSE [Syr01], GRINGO [GKKS11], I-DLV [CFPZ17]
- Stand-alone solvers: CMODELS [GLM06], SMODELS [SNS02], CLASP [GKS12], WASP [ADLR15]
- Monolithic systems: DLV [LPF+06, ACD+17], CLINGO [GKKS14]

Other approaches

- Lazy Grounding: GASP [DDPR09], ASPERIX [LN09, LBSG17], OMIGA [dCHM12], ALPHA [Wei17]
- Translation-based systems: LPTOSAT [Jan06]
- ML-based approaches analyse the grounding to select the best solver [MPR14, CDF+20]

Thanks for your attention :)

Questions?

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